PRACTICAL MODELING OF BUCKET-BRIGADE DEVICE CIRCUITS

Colin Raffel
CCRMA, Stanford University
Stanford, California, USA
craffel@ccrma.stanford.edu

Julius Smith
CCRMA, Stanford University
Stanford, California, USA
jos@ccrma.stanford.edu

ABSTRACT

This paper discusses the sonic characteristics of the bucket-brigade device (BBD) and associated circuitry. BBDs are integrated circuits which produce a time-delayed version of an input signal. In order to reduce aliasing, distortion, and noise, BBDs are typically accompanied by low-pass filters and compander circuitry. Through circuit analysis and measurements, each component of the BBD system can be accurately modeled.

1. INTRODUCTION

1.1. BBD Topology

The bucket-brigade device was invented in 1968 by Sangster at Phillips as an attempt to realize a time delay in an analog circuit [1]. BBDs are given their name because of the similarity in their functionality to a line of people passing buckets of water to fight a fire. The input signal is sampled in time and passed into a series of typically thousands of capacitors and MOS transistor switches. The charge in each capacitor stage is passed into the subsequent stage at a rate determined by an external clock signal [2]. Early attempts at implementing this architecture were unsuccessful due to inefficient transfers of charge between each stage. However, schemes using two out-of-phase clock signals developed by Sangster [3] and Reticon Corporation [2] allowed the device to be used practically in audio systems. In this method, each charge-holding element is separated by a DC-biased gate, which greatly improves transfer efficiency. This topology, which is the only one ever mass-manufactured, is shown in Figure 1. Thanks to these improvements, the BBD effectively functions as a time-delay system that falls into the uncommon category of discrete-time, but non-digital circuitry. Interestingly, the bucket-brigade delay concept was conceived and implemented after purely digital delays, but remained a cheaper and more easily implemented device until the 1980s, when they were largely replaced by digital systems.

Figure 1: Antiphase-clock bucket-brigade circuit found in most BBD integrated circuits.

1.2. Common Circuit Architectures

During their period of widespread use, BBDs [1] were typically found in circuits implementing echo, chorus, vibrato, and flanging effects [2]. In a BBD-based echo, the input signal is passed through a bucket-brigade delay line with a variable clock frequency. Altering the clock frequency changes the time necessary for the signal to propagate through the bucket stages. The output of the delay line is then often fed back into the input with variable gain to allow for trailing echos. This architecture is shown in Figure 2.

Figure 2: Topology of a BBD-based echo circuit.

In a chorus, vibrato or flanger circuit, the delay time is varied automatically by a low-frequency oscillator. This slowly varies the phase (and at high delay-change rates, perceived pitch) of the incoming signal, which produces the desired effect. The circuit architecture commonly used for BBD-based flangers and choruses is shown in Figure 3. Some less common applications include using bucket-brigade devices as reverberators, Schroeder all-pass sections [4] and discrete-time filters [5]. Due to these circuits’ unique characteristics and quality, some musicians prefer their sound, and a small number of BBDs and BBD-based products are sold today [6] [7].

1.3. Non-ideal BBD Characteristics

Despite the early improvements in BBD functionality, a great deal of precaution is typically taken when using them to ensure reasonable audio fidelity. This is largely due to transfer inefficiencies, nonlinearities, noise, and the system’s clock frequency [8]. To

\[\text{Dynamic range compression} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{Reconstruction filter} \quad \rightarrow \quad \text{Clock generation circuit} \]

Figure 3: Topology of a BBD-based chorus circuit.

\[\text{Input} \quad \rightarrow \quad \text{Reconstruction filter} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Input} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Input} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Input} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Input} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Input} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Input} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]

\[\text{Output} \quad \rightarrow \quad \text{Dynamic range expansion} \quad \rightarrow \quad \text{BBB delay line, 4096+ stages} \quad \rightarrow \quad \text{Anti-aliasing filter} \quad \rightarrow \quad \text{Dynamic range compression} \quad \rightarrow \quad \text{Clock generation circuit} \]
Prevent overloading of the circuit and to reduce noise, compander circuits are typically seen accompanying bucket-bridge devices. Companders allow a wider dynamic range to be passed through the circuit at the cost of altering the signal’s dynamics. In echo circuits, BBDs are commonly used to sample the incoming signal at audio rates, which causes audible aliasing unless the input signal is sufficiently bandlimited. For this reason, nearly all bucket-bridge circuits are preceded and followed by low-pass filters. Interestingly, the compression and expansion stages occur outside of the filter in most BBD circuits requiring companders. Because much shorter delay times are necessary for chorus, flanger, vibrato, and reverb circuits, the number of charge-passing stages in the BBD circuit is normally significantly less. This dramatically reduces the aforementioned undesirable effects. For this reason, companders are not typically used and the low-pass filters normally have minimal effect in the audio range.

2. ANTI-ALIASING AND RECONSTRUCTION FILTERS

2.1. Filtering Requirements

For any BBD, the total delay time is given by

\[ \text{Delay Time (s)} = \frac{N}{2f_{cp}} \]

where \( N \) is the number of stages in the BBD and \( f_{cp} \) is the circuit’s clock frequency \([9]\). In an echo effect, BBD systems with 4096 or more stages are typically used to create delay times on the order of hundreds of milliseconds. One common configuration is to use a 4096-stage BBD in an echo circuit to achieve a maximum of a 300 millisecond delay time. This requires a clock frequency of

\[ \frac{4096}{2(300)} = 682.6 \text{ Hz} \]

According to the sampling theorem \([10]\), in order to prevent aliasing, no signal with frequency greater than \( \frac{f_{cp}}{2} \) may be sent through the system. For this reason, in our example, the input signal must be bandlimited to 3413.3 Hz. As another example, a flanger effect typically requires a maximum of a 10 ms delay \([11]\). BBD-based flangers typically use a single, 1024-stage delay line, giving a clock frequency of

\[ \frac{1024}{2(.010)} = 51.2 \text{ kHz} \]

This requires an input signal bandlimited to 25.6 kHz. Chorus effects use similarly low delay times and high clock frequencies. The bandwidth of audio signals extends to 20 kHz, so in the case of a BBD-based echo circuit, dramatic anti-aliasing and reconstruction filters are needed before and after the delay line. Low-pass filters are also typically included in flanger and chorus circuits because the input signal could contain inaudible, high-frequency components that would be aliased into the audio range by the sampling process.

2.2. Filter Implementations

The majority of BBD-based circuits use Sallen-Key low-pass filters \([12]\) to ensure the input signal is appropriately bandlimited. A common implementation is to use a third-order filter for anti-aliasing and a third-order filter followed by a second-order “corner correction” filter for reconstruction. A typical, transistor-based Sallen-Key filter used for anti-aliasing in a BBD circuit is shown in Figure 4. The filter cutoff frequency is typically chosen to be between \( \frac{1}{2} \) and \( \frac{1}{3} \) of the sampling frequency. The specific cutoff frequency depends on the circuit designer’s preferred trade-off between aliasing distortion and high-frequency loss. In some BBD-based echo circuits, the filter cutoff can be as low as 1.5 kHz.

A less common filtering technique involves the use of switched-capacitor filters \([13]\). These filters have a cutoff frequency controlled by a clock signal, so in the case of a BBD-circuit, the clock signal is used to simultaneously set the delay time and filter cutoff frequency. In this way, the signal is only bandlimited as necessary for any given clock frequency, so that shorter delay times have a higher fidelity, in contrast to the “worst-case” fixed filter design. The Sallen-Key topology is generally preferred, however, due to its simplicity and low cost.

![Figure 3: Topology of a BBD-based chorus or flanger circuit.](image-url)

In Figure 4, the filter cutoff frequency is typically chosen to be between \( \frac{1}{2} \) and \( \frac{1}{3} \) of the sampling frequency. The specific cutoff frequency depends on the circuit designer’s preferred trade-off between aliasing distortion and high-frequency loss. In some BBD-based echo circuits, the filter cutoff can be as low as 1.5 kHz.

![Figure 4: Typical third-order Sallen-Key anti-aliasing filter used in a BBD circuit.](image-url)

The transfer function of a second-order Sallen-Key filter is given by

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{s^2 + \frac{R_1}{RC_1} + \frac{1}{C_2}} \]

where all resistances are equal to \( R \), and where \( C_1 \) and \( C_2 \) denote the two capacitor values used. The third-order case is similar, with a transfer function given by

\[ \frac{V_{out}(s)}{V_{in}(s)} = \frac{b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]

Interestingly, the vast majority of BBD circuits use transistor-based Sallen-Key circuits, but op-amp based filters are not uncommon.
where

\[
\begin{align*}
    b_0 &= a_0 = \frac{1}{R_1C_1C_2C_3} \\
    a_1 &= \frac{1}{RC_1C_2} + \frac{1}{RC_2C_3} \\
    a_2 &= \frac{2}{RC_1} + \frac{2}{RC_2} \\
    a_3 &= 1
\end{align*}
\]

and as above, all resistance are equal to \( R \) and \( C_1, C_2, \) and \( C_3 \) denote the three capacitor values used. As discussed previously, a typical filter configuration involves a third-order anti-aliasing filter followed by a single second-order and a single third-order reconstruction filter in series. Interestingly, the vast majority of BBD systems use \( R = 10 \) kOhm for all of the resistances in all three filters. Typical values for the capacitors for an echo circuit are \( C_1 = .0068 \) \( \mu \)F, \( C_2 = .082 \) \( \mu \)F, \( C_3 = .00033 \) \( \mu \)F for the anti-aliasing filter, \( C_1 = .039 \) \( \mu \)F, \( C_2 = .00033 \) \( \mu \)F for the second-order reconstruction filter, and \( C_1 = .0022 \) \( \mu \)F, \( C_2 = .033 \) \( \mu \)F, \( C_3 = .001 \) \( \mu \)F for the third-order reconstruction filter. The amplitude response curves for each Sallen-Key filter using these component values is shown in Figure 5.

When analyzing the filters in a BBD-based circuit, it is sufficient to ignore the effect of the BBD and treat all the filters as in series due to the fact that the non-linear elements are minimal and do not have drastic effects on the frequency response. In this way, we can calculate the effective transfer function as the product of the transfer functions of each filter \[14\], resulting in an eighth-order low-pass filter. This simplification method agrees with the measured results shown in Figure 6. To obtain the measured amplitude response shown, noise was inserted into an analog echo circuit directly before the anti-aliasing filter. The output signal was measured after the reconstruction filters and averaged over 128 values to smooth the random spectra of noise. The expected curve shown is the amplitude response of the series transfer function calculated with the measured circuit’s resistance and capacitance values. The calculation was done by inserting the calculated transfer function coefficient values into Matlab’s \texttt{freqz} function \[15\].

Assuming no audible aliasing, the combined filter response can be accurately modeled with a digital IIR filter. By first obtaining a frequency response using the methods described above, a difference equation can be found by any of a large number of filter design techniques \[14\]. When using the equation-error method implemented by the \texttt{invfreqz} function included in Octave Forge and Matlab’s Signal Processing Toolbox \[15\], a highly accurate, low-order digital IIR filter representation can be found. The main source of error in the digital filter is in the phase response at very high frequencies, but this error can be made smaller by using higher-order digital filters. Figure 7 shows the calculated frequency response shown in Figure 6 compared with an eighth-order digital representation. In this way, the filters in any BBD circuit can be accurately digitally modeled at low computational cost based solely on the resistance and capacitance values used in the circuit. A Matlab script for calculating filter coefficients based on filter component values can be found at the webpage for this project. If the component values are not available, it is also effective to directly model the measured frequency response of the circuit. This measurement can also serve as a check to the resistance and capacitance value-based calculation.

For circuits in which the Nyquist limit \[13\] of the BBD’s sampling process is lower than 20 kHz, it is particularly important to accurately model the low-pass filters because their effect on the delayed signal’s timbre can be dramatic. For circuits like flangers, choruses, and vibratos in which the clock frequency is significantly higher, the filter cutoff frequency may be above the Nyquist limit of the sampled signal in the modeled digital system. In this case, the filters can be neglected. However, it is common to find audio-range low-pass filters in shorter delay time circuits, so in order to ensure an accurate tonality, the filters should be included in the model.

Figure 5: Typical magnitude responses of anti-aliasing filter and third- and second-order reconstruction filters.

Figure 6: Measured and expected amplitude response curves for the series combination of anti-aliasing and reconstruction filters.

Figure 7: Measured and expected amplitude response curves for the series combination of anti-aliasing and reconstruction filters.
3. COMPRESSION AND EXPANSION

3.1. The Dynamic Range of BBDs

Transfer inefficiencies and nonlinearities in the BBD result in a system with a limited dynamic range [9]. In order to obtain a low total harmonic distortion level, the input signal level cannot exceed a small fraction of the supply voltage level. For near-maximum-level inputs, the BBD system typically has a signal-to-noise ratio of about 60 dB. This requires that the input signal be close to the maximum allowed level in order to achieve a low output noise ratio. Additionally, the BBD system has a variable insertion gain which varies between 0 and 2 dB in most implementations.

The combination of these effects results in a system which performs best with signals near the maximum input level. Because these effects are more dramatic as the number of stages increase and vary with the clock frequency, much more compensation is required in highly variable, long-delay systems such as an echo circuit. In systems with shorter delay times and a clock frequency that does not vary dramatically, such as chorus, flanger, and vibrato circuits, these effects are generally ignored.

3.2. Companding

Compensation for the suboptimal characteristics of the bucket brigade device is typically done by using a compander circuit. Companding is very common when a signal is sent through a channel with limited dynamic range. As the name suggests, a compander circuit consists of a compressor and an expander. The compressor is used before the BBD to lower the dynamic range of the incoming signal. Then, the BBD’s output is expanded in order to retain the original signal’s dynamic characteristics. To achieve minimal total harmonic distortion with a maximal signal to noise ratio, the compander is typically configured to send a signal with a near-maximum level through the BBD. The compander’s effect is visualized in Figure 7. Different BBD systems place the compression and expansion in different places in the signal chain. In some cases, the compression and expansion takes place outside of the anti-aliasing and reconstruction filters, while in others, the compander is implemented directly at the input and output of the BBD. The feedback path also varies from system to system; different circuits place it inside or outside of the compression and expansion.

![Figure 7: Combined calculated and digitized frequency responses for anti-aliasing and reconstruction filters.](image)

3.3. 570 and 571-series Companders

The majority of BBD systems that require companding utilize the 570 or 571-series compander chip. This integrated circuit consists of a pair of variable gain amplifiers and signal level averagers [16]. In BBD systems, one half of the device is used as a feedback compressor while the other half is used as a feedforward expander.

The 570 or 571 compander chip is internally set so that each amplifier has a compression or expansion ratio of 2 [17]. The time constant of the averaging circuit in the compressor or expander is determined by an external capacitor. This capacitor sets the time constant of the chip’s rectifier circuit along with an internal 10 kOhm resistor. In this way, the decay time of the circuit is given by

$$\tau = 10000C_{\text{rect}}.$$  

As in most analog averaging systems, the value of the capacitor is chosen to smoothly average the input signal without any “ripple” caused by the circuit following the instantaneous signal level at low frequencies [18]. In BBD systems, the capacitor value is normally chosen to be between 0.22 uF and 1 uF.

3.4. Compander modeling

Modeling of a compander circuit can easily be achieved by using the average signal level to determine the gain of a system [17]. If the output gain is directly proportional to the average input level, the system will act as an expander, while a system in which the gain is inversely proportional to average output signal level will compress the dynamic range. To achieve the gain ratio of two apparent in 570 and 571-series companders, the gain should be exactly proportional. In other words, the feedforward expander should be implemented as

$$f(x) = \text{avg}(|x|)x$$

while the feedback compressor should be modeled as

$$f(x) = \frac{x}{\text{avg}(|f(x)|)}.$$  

As mentioned previously, the input $x$ for each equation depends on the architecture of the BBD system being modeled.

The 570 and 571-series compander finds the average signal level by full-wave rectifying the signal and then passing it through

$$\text{avg}(|x|) = \frac{1}{T} \int_{0}^{T} |x(t)| dt.$$  

4The 571 and 570 varieties of this chip are functionally equivalent, and are manufactured by a number of different companies. As a result, any of NE- SA- or V- 570 or 571 will be found in BBD-based circuits, all performing equivalently. The 572 chip is another compander, but is slightly different internally and less common.
the RC circuit described above. A highly accurate way to model this averaging circuit is to use a one-pole digital filter. Any RC low-pass filter can be accurately modeled by

\[ y(n) = x(n) \frac{T}{RC + T} + y(n-1) \frac{RC}{RC + T} \]

where \( T \) is the sampling interval of the digital system. In the case of the averager, the input should be the absolute value of the signal to simulate full-wave rectification. In this way, the simplicity of the compander allows accurate modeling at low computational cost.

4. Bucket-Brigade Devices

4.1. Modeling BBD aliasing

As discussed previously, a variety of effects prevent BBDS from realizing an ideal time delay. The most significant effect is aliasing due to the discrete-time sampling of the input signal. Because BBDS are typically paired with anti-aliasing and reconstruction low-pass filters which ideally prevent any audible aliasing distortion, aliasing simulation is sometimes left out of BBD circuit models. The most simple implementation uses a digital delay line in place of the bucket-brigade chip. Because varying the clock frequency of the BBD alters the effective playback rate of the sampled audio, an interpolating delay line could be used to replicate this effect and to ensure a continuously variable delay time. In order to further match the characteristics of a BBD, the input to the delay line should be downsampled and the output should be upsamped according to the delay time.

The most accurate implementation would involve a software delay line of fixed length matching the number of delay stages in the BBD circuit being modeled. This delay line would take in a new input sample and output a delayed sample at a rate determined by an artificial clock frequency. Because the artificial clock frequency and sampling frequency are not necessarily related, some interpolation is needed to ensure that the correct signal level is represented in the delay line. In this way, the input signal would be appropriately downsampled and aliased according to the clock frequency. In addition, if the clock frequency changed, the samples stored in the delay line would be output at a new rate, achieving the desired pitch-change effect. This resampling technique has also been used to simulate the aliasing present in digital systems with low sampling rates. The delay line method should be chosen depending on the necessary accuracy of the model and whether or not the clock frequency of the modeled BBD system would create aliasing distortion in the audio range.

4.2. Frequency-dependent Insertion Gain

Apart from aliasing distortion, the constant transfer of charge between thousands of capacitors results in additional unintentional imperfections. One relatively minimal effect is a frequency-dependent insertion gain. This gain effectively acts as a low-pass filter with a cutoff frequency determined by the clock rate of the BBD. This effect can be thought of as a result of the BBD’s capacitors not being able to react to changes in the input signal which occur at a frequency close to the Nyquist limit. As a result, the extremity of this filtering varies with the number of clock stages. In general, the insertion gain tends to be between 0 and 2 dB at low frequencies, but falls off slowly to between -4 and -6 dB at the Nyquist limit for any given clock rate.

Because the anti-aliasing and reconstruction filters are relatively extreme and are set to have a cutoff frequency below the Nyquist limit for the lowest possible clock rate, this insertion filtering effect can be largely ignored. However, an accurate model would include this filtering in the frequency response used to generate the digital representation of the anti-aliasing and reconstruction filters. Direct measurement of this effect is difficult due to the fact that the output signal of the BBD is sampled in time. As a result, a safe method for determining the total modeled filter response involves measuring the response at the output of the reconstruction filters. An applicable insertion filtering measurement is typically graphed in the datasheet for any BBD in question.

4.3. Modeling BBD Noise

With the inclusion of a compander circuit in BBD systems with a high number of delay stages, dynamic range overloading and noise insertion can be reduced, but not completely removed. A truly accurate model would involve the insertion of low-amplitude noise immediately before or after the delay line. The appropriate noise level for a specific BBD can be measured or found in the device’s datasheet. Adding colored noise to a system is similarly done when modeling the sound quality of old recordings.

One occasional unintended feature of analog delay circuits is their ability to oscillate when the feedback loop gain is high enough. The presence of noise in the circuit allows for oscillation without external input. Adding noise to the simulated BBD delay line realizes this effect in digital models. However, in many cases the purpose of modeling a system is to create a noise-free version, so in this case adding noise would be inappropriate. Additionally, the noise level tends to be at least 60 dB below the input signal level, and is ideally further suppressed by the compander, so it can be reasonably treated as imperceptible.

4.4. BBD Nonlinearities

Despite the compander’s ability to mostly eliminate noise, a significant nonlinearity is typically present in BBD systems with a large number of stages. As an estimate, the unavoidable total harmonic distortion of an \( N \)-stage BBD can be given by

\[ \text{THD} = 1.01^{N/1024} - 1 \]

In other words, about 1% of harmonic distortion occurs for every 1024 stages of bucket-brigade delay. This figure agrees with measured results as well as device datasheets. An important aspect of this nonlinearity is that it does not vary greatly depending on the signal level—in other words, it is not a clipping distortion. Based on this estimate, it is apparent that modeling this nonlinearity is less important for choruses, flangers, and vibratos, which tend to use BBDS with 1024 stages or less, and it is typically left out. However, for echo circuits, in which the delayed signal is constantly recycled through the delay line, this distortion can quickly accumulate and become an audible component of the system.

An effective way to characterize the distortion present in this nonlinearity is to measure the spectrum at the output of the reconstruction filters for a pure sine-wave input at various frequencies and amplitudes. The aliasing present before the reconstruction filters results in unreliable data, and it is safe to assume that the transistor-based Sallen-Key filters are linear for the low amplitude signals output by the BBD. As a result, any additional harmonic distortion can be reasonably treated as imperceptible.
content present can be assumed to be due to the bucket-bridge
device.

A typical measured spectrum for a BBD system with thou-
sands of stages is shown in Figure 9. This spectrum suggests
that the added harmonic components fall off linearly in magnitude,
with the fourth and higher harmonics at almost imperceptibly low
magnitude. This characteristic holds across all frequencies that are
within the passbands of the low-pass filtering apparent in a BBD
system. The characteristics of the measured nonlinearity are also
similar for a range of amplitudes. The peaks of the BBD output
spectrum for a sine wave input at various amplitudes is shown in
Figure 10. These spectra confirm that the added harmonic compo-
nents decrease linearly in magnitude.

4.5. Nonlinearity Modeling

Accurate modeling of this nonlinearity is difficult because it does
not largely depend on signal amplitude. Most easily implemented
nonlinearity models add much more harmonic content as the signal
level increases [11]. However, a reasonably good estimate can be
achieved by using a third-order polynomial nonlinearity, given by

\[
f(x) = \begin{cases} 
1 - a - b & \text{for } x > 1 \\
x - ax^2 - bx^3 + a & \text{for } -1 < x < 1 \\
-1 - a + b & \text{for } x < -1 
\end{cases}
\]

where \( a \) and \( b \) are parameters set in order to match the measured
BBD output spectrum for a pure sine wave input. The inclusion of
the \( a \) constant in the \(-1 < x < 1\) case ensures that the signal will
average around 0. The boundary or clipping cases are formulated
so that the nonlinearity smoothly transitions between potential in-
put signal ranges. The output spectrum of an attempted match to
the BBD-based distortion shown in Figure 10 using coefficients
of \( a = \frac{1}{8} \) and \( b = \frac{1}{18} \) is shown in Figure 11. It is apparent that
the spectrum matches the measured result for high amplitudes but
underestimates the level of added harmonics as the amplitude de-
creases. The transfer function for the nonlinearity corresponding
to these coefficients is plotted in Figure 12.

![Figure 9: Measured output spectrum of a BBD with sine wave input.](image)

![Figure 10: Peaks in output spectra of a BBD with a sine wave input at various amplitudes.](image)

![Figure 11: Peaks in output spectra of a third-order polynomial nonlinearity with coefficients set to match the harmonic distortion characteristics of a BBD system.](image)

![Figure 12: Waveshaping transfer function for nonlinearity with coefficients set to match the characteristics of a BBD system.](image)
The inaccuracies at low amplitudes are made less relevant by the presence of the simulated compression and expansion which ensures that the input to the nonlinearity is consistently fairly large. This causes the error in the harmonics’ amplitudes to be smaller in general in a fully modeled system. A better nonlinearity approximation could be obtained by altering the values of the polynomial coefficients according to the average signal level. Another method could calculate and add the harmonics in the frequency domain, at the cost of computational power and accuracy. Finally, an amplitude-dependent, multiple-table-lookup system could be used to match the characteristics of a specific circuit. One convenient aspect of the BBD system is that the input signal will be bandlimited by the anti-aliasing filter before being distorted by any nonlinearity. This avoids the aliasing effects that commonly arise when using polynomial nonlinearities in discrete-time systems 11.

5. CONCLUSION

The characteristics and modeling of bucket-brigade devices and their typical companion circuitry was discussed. In particular, it was shown that a highly accurate model of the requisite anti-aliasing and reconstruction filters can be found by using a lower-order digital IIR filter based on the resistance and capacitance values of the filters, under the assumption of no audible aliasing. It was also shown that the combadry circuitry that accompanies BBDs with many stages can be modeled with a pair of amplifiers and a pair of averaging functions with an averaging time determined by the circuit’s rectifier capacitor value. The unexpected effects of BBDs were discussed, and methods were proposed for modeling the corresponding nonlinearities. The importance of the component of the model relative to the system’s delay time was compared. In particular, it was shown that for short-delay circuits such as choruses, flangers, and vibratos, many aspects of the model can be omitted, but for longer-delay devices such as echo systems, a detailed model is necessary to ensure a perceptually equivalent output signal.

An example model of a BBD-based echo circuit using the Synthesis ToolKit 22 can be found at the webpage for this project.

6. ACKNOWLEDGMENTS

We would like to thank Adam Sheppard and Travis Skare for lending BBD-based circuits for measuring and verifying results. David Yeh helped identify the topology of the anti-aliasing and reconstruction filters. Thanks to Jonathan Abel for his suggestions on BBD imperfections and help with filter modeling.

7. REFERENCES


https://ccrma.stanford.edu/~craffel/software/bbdmodeling/BBDfilter.m