Learning Hard Alignments with Variational Inference

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Abstract

There has recently been significant interest in hard attention models for tasks such as object recognition, visual captioning and speech recognition. Hard attention can offer benefits over soft attention such as decreased computational cost, but training hard attention models can be difficult because of the discrete latent variables they introduce. Previous work has used REINFORCE and Q-learning to approach these issues, but those methods can provide high-variance gradient estimates and be slow to train. In this paper, we tackle the problem of learning hard attention for a 1-d temporal task using variational inference methods, specifically the recently introduced VIMCO and NVIL. Furthermore, we propose novel baselines that adapt VIMCO to this setting. We demonstrate our method on a phoneme recognition task in clean and noisy environments and show that our method outperforms REINFORCE with the difference being greater for a more complicated task.

1 Introduction

Attention models have gained widespread traction from their successful use in tasks such as object recognition, machine translation, image captioning, speech recognition, etc... [16, 3, 20, 5]. Attention is typically used to integrate information from different parts of the input before producing outputs. For example, in the case of object recognition, this might be different patches of the input image, for translation this might be different words or phrases in the source sentence, and for speech recognition this might be different regions of the input speech utterance.

Soft attention integrates information from the entire input space and can be slow because it incurs a cost proportional to the size of the input for each output produced. Hard attention does not incur such a cost, but during training require marginalizing over the latent variables corresponding to the location of attention. Since these are discrete, methods for learning with discrete latent variables are required, with the REINFORCE [19] algorithm being the default choice. Soft attention models are fully differentiable and hence do not face this problem. Nevertheless, the need for explicitly integrating attention over entire input seems inherently sub-optimal, especially since most machine learning tasks such as vision, speech, and language have significant local structure that should be useful for guiding attention mechanisms. As a result, it is important to explore alternative methods for training hard attention models.

One alternative method for training discrete latent variable models is variational inference, which trains a second model, called the variational posterior, that approximates the true posterior over the

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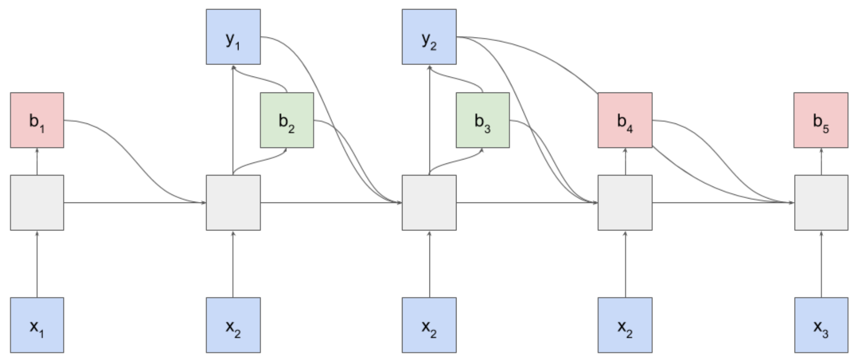
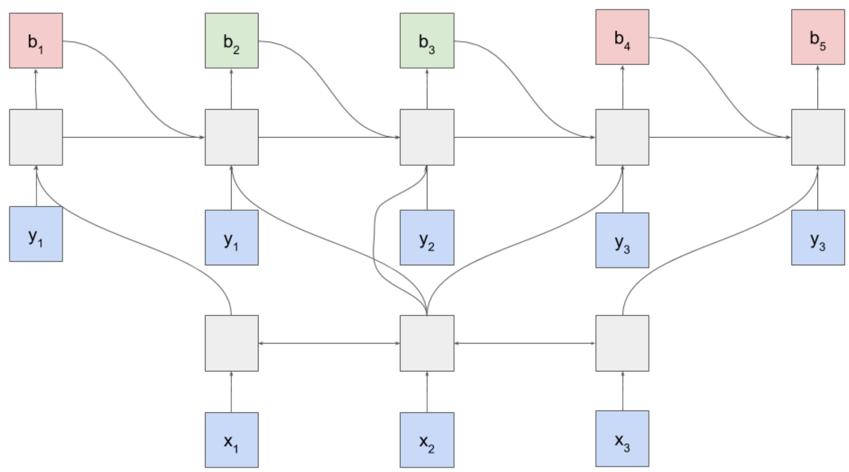


(a) Model



(b) Variational Posterior

Figure 1: A diagram of our models. (a) Shows the model we are training and (b) shows the model of the posterior, which refer to as the variational posterior. Blue boxes represent inputs and outputs. Red and green boxes represent the latent alignment variables, with red meaning that the model chose to consume an input and not emit an output and green meaning that the model chose to produce an output and not consume and input. For example, in figure (a) note that at timestep 1, b_1 is red so at the next timestep x_2 is fed in and the output is compared against y_1, i.e. the input is advanced but the output is not advanced. In training the model, samples are generated from the variational posterior and fed into the model and the variational objective is optimized to adjust both models jointly.

latent variables. The variational posterior can incorporate information from the observed variables when assigning probabilities to the latent variables, which can make approximating the intractable integral over the latent variables much more efficient. In [15] and [14] the authors show that variational methods can outperform REINFORCE and other algorithms for learning in stochastic networks. In this paper we leverage these recent developments as an alternative to REINFORCE for fitting hard attention models.

We demonstrate our method, which specializes the VIMCO algorithm [15] to the temporal setting, by using it to train online sequence-to-sequence models [11, 13]. Online sequence-to-sequence models are sequence-to-sequence models that emit tokens incrementally as input arrives, instead of waiting for the entire input to be received. This opens up the possibility for using sequence-to-sequence models for real world tasks such as speech recognition and online translation, where one may not want to wait for the entire input to be received before output can be produced. These models have previously been trained using a Viterbi-like training procedure [11], and with REINFORCE [13] and have shown to be promising models for speech recognition. While the former paper shows strong results, it was applied in a specific setting where the inputs were chunked into local blocks, where as the latter paper was applied to a more general, unconstrained set up.

In summary, the contributions of this paper are as follows:

1. We apply recent variational inference algorithms to learning hard attention models in a temporal setting with discrete latent variables, and demonstrate improvements in performance and training time over the standard REINFORCE approach.
2. We introduce a new baseline that adapts VIMCO to the temporal setting.
3. We demonstrate our methods on two phoneme recognition tasks, TIMIT and a noisy multi-speaker version of TIMIT which we call Multi-TIMIT.

2 Methods

2.1 Model

In this paper we use the online sequence-to-sequence model described in [13] to demonstrate our methods. Refer to the top part of Figure 1 for this section. The model receives inputs $x = x_1, \ldots, x_m$ and produces a sequence of tokens $y = y_1, \ldots, y_n$, as outputs. A token is produced by the model at a time step t only when the hard-attention variable 'fires', i.e., $b_t = 1$ (Figure 1). This hard-attention variable and the neural network state are all conditioned on the past inputs, past outputs and the past attention decisions. This lends the model great flexibility, at the cost of increased training complexity. Further, when a hard attention variable b_t fires at a given time step, the same input is fed into the model at the next time step. For example, in the figure, at time step 3, the input to the model is x_2, because the hard attention variable b_2 at step 2 fired.

Specifically, we model $p(y, b|x)$ where $y = y_1, \ldots, y_n$ is a sequence of target tokens and $x = x_1, \ldots, x_m$ is a sequence of observed inputs. The binary latent variables $b = b_1, \ldots, b_{m+n}$ define where the attention turns on and tokens are output. i.e., at time step t, if $b_t = 1$ then the model emitted an output and if $b_t = 0$ it did not. Also, if $b_t = 1$, the attention model forces the model to dwell at the same input at the next time step. Let n be the number of target tokens, m the number of inputs, and $T = m + n$ the number of steps the model is run for. Our model assumes $p(y, b|x)$ factorizes as

$$p(y, b|x) = \prod_{t=1}^{T} p(y_{O(t)}|b_{1:t}, x_{1:I(t)}, y_{1:O(t-1)})^{b_t} p(b_t|b_{1:t-1}, x_{1:I(t)}, y_{1:O(t-1)})$$

where $O(t) = \sum_{i=1}^{t} b_i$ is the position in the output at time t and $I(t) = 1 + \sum_{i=1}^{t-1}(1 - b_i)$ is the input position at time t.

To reduce notational clutter at the risk of ambiguity, from here on, we will use y_t to implicitly mean $y_{O(t)}$ (i.e., the target at step t). Similarly, we will refer to $x_{I(t)}$ as x_t and similarly for ranges over time for these variables.

2.2 Learning

Previous approaches to fitting the model [13] used REINFORCE [19] with a parametric baseline. We briefly summarize this approach and note that it optimizes a lower bound on the log-likelihood. In [13] Luo et. al. optimize

$$\mathbf{E}_b\left[\sum_{t=1}^{T} b_t \log p(y_t|s_t, b_t)\right] = \mathbf{E}_b\left[\log \prod_{t=1}^{T} p(y_t|s_t, b_t)^{b_t}\right] \leq \log p(y|x),$$

where $s_t = \{b_{1:t-1}, x_{1:t}, y_{1:t-1}\}$ is the state at time t and the expectations are over $p(b_t|s_t)$. Differentiating this objective gives

$$\mathbf{E}_b\left[\nabla \sum_{t=1}^{T} b_t \log p(y_t|s_t, b_t)\right] + \sum_{t=1}^{T} \mathbf{E}_b\left[\left(\sum_{t' \geq t}^{T} R_{t'}\right) \nabla \log p(b_t|s_t)\right],$$

where $R_t = b_t \log p(y_t|s_t, b_t)$ is the reward at timestep t. The first term can be effectively estimated with a single Monte Carlo sample, while the second term exhibits high variance. To reduce the variance, we can subtract a learned baseline $c(b_{1:t}, x_{1:T}, y_{1:T})$ from the sum of rewards, which does not change the expectation as long as it is independent of b_t. They also find that adding a regularization term to maximize entropy is necessary to achieve good performance.

Performing stochastic gradient ascent with this gradient estimator is the standard REINFORCE algorithm where the reward is the log-likelihood. Unfortunately, this algorithm requires sampling b_t from $p(b_t|s_t)$ during training, which can lead to gradient estimates with high variance when settings of b that assign high likelihood to y are rare [15]. To address this, we introduce a variational posterior (see figure 1 (b) for the neural architecture we use for this posterior)

$$q(b|x,y) = \prod_{t=1}^{T} q(b_t|b_{1:t-1}, x_{1:T}, y_{1:T}). \tag{1}$$

The variational posterior has access to all x, y, and past b, thus it can leverage this additional information to assign high probability to b that produce large values of $p(y|b,x)$. Intuitively, in speech recognition, knowing the token the model must emit is helpful when deciding when to emit. Using q we can derive a similar lower bound on the log-likelihood

$$\log p(y|x) = \log \mathbf{E}_{b\sim q}\left[\frac{p(y,b|x)}{q(b|x,y)}\right] \geq \mathbf{E}_{b\sim q}\left[\log \frac{p(y,b|x)}{q(b|x,y)}\right].$$

where we can simultaneously optimize q and the parameters of the model to improve the lower bound. To maximize the lower bound, we use the variance reduction techniques introduced in [14]. Setting $q(b|x,y) = \prod_t p(b_t|s_t)$ recovers the REINFORCE objective.

2.2.1 Multi-sample Objectives

Both the REINFORCE and the variational inference objectives admit multi-sample versions that give tighter bounds on the log-likelihood [4]. In particular, the multi-sample variational lower bound is

$$\mathcal{L} = \mathbf{E}_{b^{(1:k)}\sim q}\left[\log\left(\frac{1}{k}\sum_{i=1}^{k}\frac{p(y,b^{(i)}|x)}{q(b^{(i)}|x,y)}\right)\right]$$

where k is the number of samples and $b^{(i)}$ denotes the ith sample of the latent variables. The gradient of this lower bound is

$$\mathbf{E}_{b^{(1:k)}\sim q}\left[\sum_{i=1}^{K}\sum_{t=1}^{T}\mathcal{L}\nabla\log q(b_t^{(i)}|x,y) + \frac{f^{(i)}\nabla\log f^{(i)}}{\sum_{j=1}^{K} f^{(j)}}\right],$$

where $f^{(i)} = \frac{p(y,b^{(i)}|x)}{q(b^{(i)}|x,y)}$. Setting $q(b|x,y) = \prod_t p(b_t|s_t)$ recovers the multi-sample analogue to REINFORCE. Similarly to the REINFORCE objective, we can subtract a baseline $c(b_{1:t-1}^{(i)}, x_{1:T}, y_{1:T}, b_{1:T}^{(-i)})$ from $\mathcal{L}$ that does not depend on $b_t^{(i)}$ without changing the expectation. In [15], the authors study a similar objective and show how to estimate it with Monte Carlo samples.

2.3 Variance Reduction

Training these models is challenging due to high variance gradient estimates. We can reduce the variance of the estimators by using information from multiple samples to construct baselines. In particular, for REINFORCE, we can write the gradient update as

$$\mathbf{E}_{b^{(i)}}\left[\left(\sum_{t'\geq t}^{T} R_{t'}^{(i)} - c(s_{t-1}^{(i)}, \{R_{1:T}^{(j)}\}_{j\neq i})\right)\nabla\log p(b_t^{(i)}|s_{t-1}^{(i)})\right],$$

where c is a baseline for sample i that is a function of the ith trajectory's state up to time $t-1$ as well as the rewards produced by all other trajectories. The goal is to pick a c that is a good estimate of the sum of future rewards, and a straightforward choice of c is the average sum of future rewards from the other samples

$$c = \frac{1}{k-1}\sum_{j\neq i}\sum_{t'\geq t}^{T} R_{t'}^{(j)}.$$

This ignores the fact that $s_t^{(i)} \neq s_t^{(j)}$, which can make this standard baseline unusable. For example, in our setting different trajectories may have emitted different numbers of tokens on a given timestep, resulting in substantial differences in cumulative future rewards between trajectories that do not indicate the relative merit of those trajectories. Ideally, we would average over multiple trajectories starting from $s_t^{(i)}$, but this is computationally expensive. We can add a residual term to address this,

$$c = \frac{1}{k-1} \sum_{j \neq i} \sum_{t' \geq t}^{T} R_{t'}^{(j)} + \frac{1}{k-1} \sum_{j \neq i} \sum_{t' < t} R_{t'}^{(j)} - R_{t'}^{(i)} \tag{2}$$

but this results in a baseline that is the same across all timesteps, thus potentially increasing variance as all decisions are rewarded or punished together. We will call this the *leave-one-out* baseline because the baseline for a given sample is constructed using an average of the return of the other $k-1$ samples.

As the sum of future rewards strongly depends on the number of emitted tokens at time t, we can sum the future rewards in the other samples starting from when they have emitted the same number of tokens as sample i. In particular, let $e_t^{(j)} = \min_{t'} p_y^{(j)}(t') \geq p_y^{(i)}(t)$ be the first timestep when sample j has emitted the same number of tokens as sample i at timestep t, then

$$c = \frac{1}{k-1} \sum_{j \neq i} \sum_{t' > e_{t-1}^{(j)}}^{T} R_{t'}^{(j)}. \tag{3}$$

We call this the *temporal leave-one-out* baseline because it takes into account the temporal structure of our setting. These baselines can be combined with the parametric baseline, and all of these baselines are applicable to both the variational inference and REINFORCE objectives.

In summary, we covered objectives that vary along two axes: the presence or absence of a variational posterior and the use of multiple samples vs a single sample. Both of these axes offer promising avenues for variance reduction reduction, but the sequential setting has a unique reward structure that make the best course unclear. Finally, we presented a novel baseline adapted to our setting.

3 Related Work

In this section we first highlight the relationship between our model and other models for attention. Tang et. al. [18] proposed visual attention within the context of generative models, while Mnih et. al. [16] proposed using recurrent models of visual attention for discriminative tasks. Subsequently, visual attention was used in an image captioning model [20]. These forms of attention use discrete variables for attention location. Recently, 'soft-attention' models were proposed for neural machine translation and speech recognition [3, 6]. Unlike the earlier mentioned, hard-attention models, these models pay attention to the entire input and compute features by blending spatial features with an attention vector that is normalized over the entire input. Our paper is most similar to the hard attention models in that features at discrete locations are used to compute predictions. However it is different from the above models in the training method. While the hard attention models use REINFORCE for training, we follow variational techniques. We are also different from the above models in the specific application – attention in our models is over temporal locations only, rather than visual and temporal locations.

Because the attention model we use is hard-attention, the model we use has parallels to prior work on online sequence-to-sequence models[11, 13]. The neural transducer model can use either hard attention, or a combination of hard attention with local soft attention. However it has a specific block based architecture, and it is trained with an approximate maximum likelihood procedure that is similar to a policy search. The model of Luo et. al. [13] is most similar to our model. Both models use the same architecture; however, while they use REINFORCE for training, we explore VIMCO for training the attention model. A similar model with REINFORCE has also been used for training an online translation model[10] and for training Neural Turing Machines [22]. Our work would be equally valid for these domains, in future work.

There has also been work using reweighted wake sleep to train sequential models. In [1], Ba et. al. optimize a variational lower bound with the prior without using a variational posterior. In this

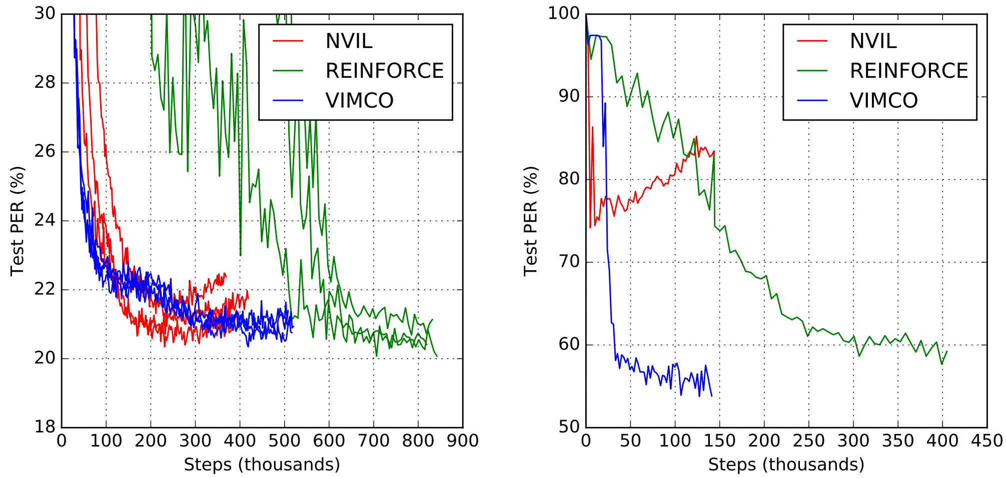


Figure 2: Training Curves: This figure shows the training curves for the different methods explored in this paper. Figure on left shows the results on the TIMIT dataset (three trials), while the figure on the right show the results on the Multi-TIMIT dataset. It can be seen that VIMCO requires a much smaller number of steps compared to REINFORCE in both cases. NVIL can perform well enough on a simple task like TIMIT, but struggles with Multi-TIMIT. It can be seen that the gap between REINFORCE and VIMCO increases with MULT-TIMIT. This is presumably because Multi-TIMIT is a much harder task, and having a good approximation to the posterior lets the model draw attention to the correct places.

work, we refer to this as REINFORCE to distinguish it from variational inference with an inference network. In [2] the authors revisit this topic, using reweighted wake sleep to train similar models. Their algorithm makes use of an inference network but does not optimize a variational lower bound on the log marginal likelihood. Instead they optimize separate objectives for the model and the inference network that produce a biased estimate of the gradient of the log marginal likelihood.

It also bears pointing out that there has been interesting work recently on using discrete latent variables in recurrent models. These include the recent work on learning sequences using GANS [21], models for learning language models using a straight through estimator [7] and recurrent models with stochastic layers[8].

4 Experiments

For our experiments on hard attention we used the standard phoneme recognition task on TIMIT. The TIMIT dataset has 3696 training utterances with 400 utterances for validation, and 182 utterances for test. The audio waveforms were processed into frames of log mel filterbank spectrograms every 25ms, with a stride of 10ms. Each frame had 40 mel frequency channels, and one energy channel; deltas and accelerations of the features were append to each frame. As a result each frame was a 123 dimensional input. The targets for each utterance where the sequence of phonemes. We used the 61 phoneme labels provided with TIMIT for training and decoding. To compute the phone error rate (PER) we collapsed the 61 phonemes down to 39 as is standard on this task[12].

Our model was a 2-layer LSTM with 256 units in each layer. For the variational posterior model we used a 6-layer LSTM with 4 bidirectional layers over the input followed by 2 unidirectional layers. Each layer had 256 units. We regularized the models with variational noise and performed a grid search over the values $\{0.075, 0.1, 0.15\}$ for the standard deviation of the noise. We also used L2 regularization and grid searched over the values $\{1 \times 10^{-5}, 1 \times 10^{-4}, 1 \times 10^{-3}\}$ for the weight of the regularization.

Table 1: Phone error rate results on TIMIT test set for various models. This shows that REINFORCE performs comparably to the variational inference methods and that our novel baselines improve training for REINFORCE. Each number is the average of three runs. Our methods are above the horizontal line, while methods from the literature are listed below it.

Method	Phone Error Rate (PER)
REINFORCE with leave-one-out (LOO) baseline	20.5%
NVIL with LOO baseline	21.1%
VIMCO with LOO baseline	20.0%
REINFORCE with temporal LOO baseline	20.0%
NVIL with temporal LOO baseline	21.4%
VIMCO with temporal LOO baseline	20.0%
Online Alignment RNN (stacked LSTM) [13]	21.5%
Neural Transducer with unsupervised alignments [11]	20.8%
Online Alignment RNN (grid LSTM) [13]	20.5%
Monotonic Alignment Decoder [17]	20.4%
Neural Transducer with supervised alignments [11]	19.8%
RNN trained with Connectionist Temporal Classification [9]	19.6%

Table 2: PER results on Multi-TIMIT for various algorithms. Each algorithm uses a leave-one-out baseline. It can be seen that for this task VIMCO outperforms REINFORCE significantly.

Method	PER
REINFORCE with LOO baseline	57.67%
NVIL with LOO baseline	74.18%
VIMCO with LOO baseline	53.83%

4.1 Multi-TIMIT

We generate a new data set by mixing a male voice with a female voice from the original TIMIT data. Half of the male and female utterances are selected as speaker 1, and pair with an utterance coming from the opposite gender as speaker 2. The wave signal of both speakers are first scaled to the same range, and then the signal scale of the speaker 2 is reduced to half before mixing the two utterances. The same feature generation method that was described above was used, resulting in a 123 dimensional input per frame. The transcript of the speaker 1 was used as the ground truth transcript for this new utterance. This data follow the same train, dev, and test specification as TIMIT, with each set having half of the original number of utterances after the pairing. As a result there are 544 female and 1304 male in the train set, 72 female and 128 male in the dev set, 32 female and 64 male in the test set.

5 Results

In this section we describe the results and try to explain the trends we observed.

Figure 2 shows a plot of the training curves for the different methods of training and the different datasets. The variational methods (VIMCO and NVIL) require many fewer training steps compared to REINFORCE on both datasets. Notably, all methods used the same batch size and number of samples, so training steps are comparable. NVIL can perform well enough on a simple task like TIMIT, but struggles with Multi-TIMIT. It can be seen that the gap between REINFORCE and VIMCO increases with MULT-TIMIT (also see table 2). We discuss reasons for this further in section 5.1

Figure 3 shows a summary of how the REINFORCE models compare with the VIMCO models on an example utterance in TIMIT, and the corresponding noisy version of the same utterance in Multi-TIMIT. It can be seen from the figure that REINFORCE attempts to attend to the state of the RNN until more information has come in, compared to VIMCO. This is presumably because it requires more information during learning. VIMCO on the other hand gets learning signal from the variational posterior, which can access the future and find the most optimal place to attend to.

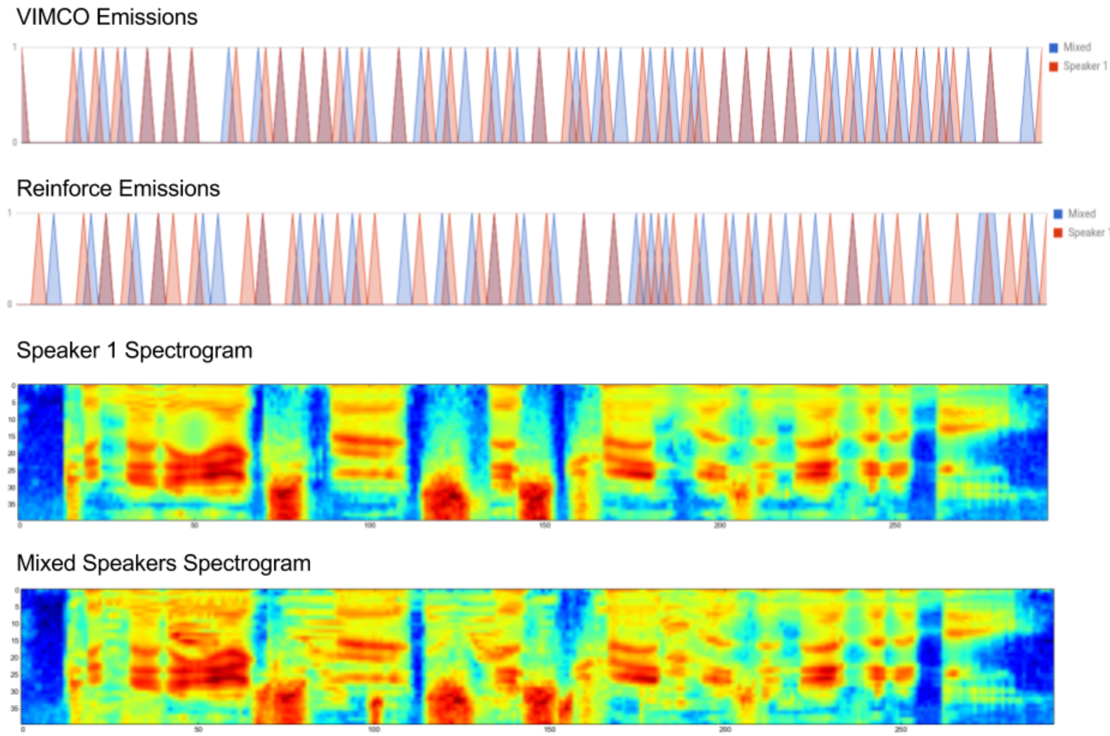


Figure 3: Emission distributions: This figure shows the probability of emitting tokens using different models, for the case of a clean utterance from TIMIT and the corresponding noisy utterance in Multi-TIMIT. It can be seen that for the Multi-TIMIT utterances, the models choose to emit tokens slightly later than they would have for TIMIT utterances.

It is also instructive to look at where the models in Multi-TIMIT attend to compared to the models in TIMIT, on the same data. It is not apparent from figure 3 whether there is a consistent trend in Multi-TIMIT to attend to later parts of the spectrogram – sometimes the Multi-TIMIT model emits later than the TIMIT models. Presumably this depends on whether there is overlapping noise at different time steps.

5.1 When does VIMCO perform much better than REINFORCE ?

In our experiments we noticed that the difference between the performance of VIMCO and REINFORCE was larger for the more complicated task of Multi-TIMIT than it was for the simpler task of TIMIT. This can be explained by looking at the kinds of samples of experience that the models learn from. In the simpler problem of single speaker (TIMIT), Monte-Carlo samples generated by REINFORCE have very high probabilities – there are only a small number of samples that explain the entire probability mass, and these are sampled easily by the left to right ancestral pass (in time) of the model because most of the conditional distributions have a probability of 1. These are very similar to the samples generated by the approximate posterior from VIMCO. As a result both methods perform approximately the same. In the case of Multi-TIMIT, however, the samples generated by REINFORCE are very different from the samples generated from VIMCO. This is because in the ancestral pass from left to right (in time) the probabilities for individual emissions are much lower. Thus the likelihood ‘appears’ less peaky, and a large diversity of samples is chosen that are actually not very good for learning. This is even though the posterior over emissions is very peaky! VIMCO, on the other hand does not face this problem because it samples from the approximate posterior, which is very peaked around the ‘correct’ samples of experience.

6 Conclusion

In this paper we have showed how we can adapt VIMCO to perform hard attention for the case of temporal problems. Our method outperforms other methods of training online sequence to sequence models. The improvements are greater for more difficult problems such as noisy mixed speech. In the future we will attempt to show how such variational techniques can be used to improve hard attention on more general tasks.

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